$$\therefore\ x=\frac{100+\ \frac{5}{.03}(1.03^{10}-1)}{1.03^{10}}=\$117.0604.$$

For (b), $a=100$, $n=10$, $R=.05$, $r=.03$, $r'=.04$.

$$\therefore\ x=\frac{100+\ \frac{5}{.04}(1.04^{10}-1)}{1.03^{10}}=\$119.0777.$$

If in (a) interest were payable semi-annually, we should have $a=100$, $n=20$, $R=.025$, $r=.015$, $r'=.015$, and $x=\$117.168+$, or \$117.17 as given in the tables of bond values used by brokers and bankers.

Also solved by *E. W. MORRELL*, *B. F. YANCEY* and *G. B. M, ZERR*. Prof. Morrell obtained as results \$118.356 and \$117.661; and Proposer, to last part, \$117.60.

57. Proposed by J. C. CORBIN, Pine Bluff, Arkansas.

Find the quotient of

$$\begin{vmatrix} (s-a_1)^2 & a_1{}^2 & a_1{}^2\ldots\ldots a_1{}^2 \\ a_2{}^2 & (s-a_2)^2 & a_2{}^2\ldots\ldots a_2{}^2 \\ a_3{}^2 & a_3{}^2 & (s-a_3)^2\ldots\ldots a_3{}^2 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots \\ a_n{}^2 & a_n{}^2 & a_n{}^2\ldots s-a_n{}^2 \end{vmatrix} \div \begin{vmatrix} s-a_1 & a_1 & a_1\ldots\ldots\ldots\ldots a_1 \\ a_2 & s-a_2 & a_2\ldots\ldots\ldots\ldots a_2 \\ a_3 & a_3 & s-a_3\ldots\ldots\ldots a_3 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots \\ a_n & a_n & a_n\ldots\ldots\ldots s-a_n \end{vmatrix}$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Let Q=the quotient and as we can exchange row for column without altering the value, we get

$$Q=\begin{vmatrix} (s-a_1)^2 & a_2{}^2 & a_3{}^2\ldots a_n{}^2 \\ a_1{}^2 & (s-a_2)^2 & a_3{}^2\ldots a_n{}^2 \\ a_1{}^2 & a_2{}^2 & (s-a_3)^2\ldots a_n{}^2 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots \\ a_1{}^2 & a_2{}^2 & a_3{}^2.(s-a_n)^2 \end{vmatrix} \div \begin{vmatrix} s-a_1 & a_2 & a_3\ldots\ldots\ldots a_n \\ a_1 & s-a_2 & a_3\ldots\ldots\ldots a_n \\ a_1 & a_2 & s-a_3\ldots\ldots\ldots a_n \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots \\ a_1 & a_2 & a_3\ldots\ldots s-a_n \end{vmatrix}$$

All the elements in the i^{th} column of the numerator being $a_i{}^2$, of the denominator a_i, except in the i^{th} row which is $(s-a_i)^2$ for numerator, and $s-a_i$ for denominator. Hence, we have

$$Q=\begin{vmatrix} 1, & 0, & 0, & 0,\ldots\ldots \\ 1, & (s-a_1)^2, & a_2{}^2, & a_3{}^2,\ \ldots\ldots \\ 1, & a_1{}^2, & (s-a_2{}^2,) & a_3{}^2,\ \ldots\ldots \\ 1, & a_1{}^2, & a_2{}^2, & (s-a_3)^2,\ \ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{vmatrix} \div \begin{vmatrix} 1, & 0, & 0, & 0,\ldots\ldots \\ 1, & s-a_1, & a_2, & a_3,\ldots\ldots \\ 1, & a_1, & s-a_2, & a_3,\ldots\ldots \\ 1, & a_1, & a_2, & s-a_3,\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{vmatrix}$$

Multiply first column of numerator by $a_i{}^2$, of the denominator by a_i and subtract from the i^{th} column; do this for each column and the value is unaltered.

$$\therefore Q=\begin{vmatrix} 1, & -a_1^2, & -a_2^2, & -a_3^2, \ldots \\ 1, & s(s-2a_1), & 0, & 0, \ldots \\ 1, & 0, & s(s-2a_2), & 0, \ldots \\ 1, & 0, & 0, & s(s-2a_3), \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix} \div \begin{vmatrix} 1, & -a_1, & -a_2, & -a_3, \ldots \\ 1, & s-2a_1, & 0, & 0, \ldots \\ 1, & 0, & s-2a_2, & 0, \ldots \\ 1, & 0, & 0, & s-2a_3, \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

Let $u=(s-2a_1)(s-2a_2)(s-2a_3)\ldots\ldots(s-2a_n)$.

$$\sum\frac{a_i^2}{s-2a_i}=\frac{a_1^2}{s-2a_1}+\frac{a_2^2}{s-2a_2}+\frac{a_3^2}{s-2a_3}+\ldots\ldots$$

$$\therefore Q=\frac{s^{n-1}u\left\{s+\sum\frac{a_i^2}{s-2a_i}\right\}}{u\left\{1+\sum\frac{a_i}{s-2a_i}\right\}}=\frac{s^{n-1}\left\{s+\sum\frac{a_i^2}{s-2a_i}\right\}}{\left\{1+\sum\frac{a_i}{s-2a_i}\right\}}.$$

ERRATA. On page 52 of last issue, line 3 from bottom, read $=$ before $\frac{1}{c}$, and in the denominator read $\sqrt{a^2-x^2}$ for "$\sqrt{a^2+x^2}$"; on page 53, line 15, extend the radical sign over a^2-x^2 and b^2-x^2, in the numerators.

PROBLEMS.

64. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Solve the equations:

$$a^2x=(2x^2-a^2)\sqrt{x^2+y^2}\ldots\ldots(1).$$
$$b^2y=(2y^2-b^2)\sqrt{x^2+y^2}\ldots\ldots(2).$$

65. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Prove that $\cos\frac{n\pi}{7}+\cos\frac{3n\pi}{7}+\cos\frac{5n\pi}{7}=\frac{1}{2}$ or $-\frac{1}{2}$, according as n is *odd* or *even*.